6-1. Draw the influence lines for (a) the moment at $C$, (b) the reaction at $B$, and (c) the shear at $C$. Assume $A$ is pinned and $B$ is a roller. Solve this problem using the basic method of Sec. 6-1.
(a)

(b)

(C)


6-2. Solve Prob. 6-1 using the Müller-Breslau principle.

(a)

(b)

(C)


6-3. Draw the influence lines for (a) the vertical reaction at $A$, (b) the moment at $A$, and (c) the shear at $B$. Assume the support at $A$ is fixed. Solve this problem using the basic method of Sec. 6-1.
(a)

(b)

(c)

*6-4. Solve Prob. 6-3 using the Müller-Breslau principle.


(b)

(C)


6-5. Draw the influence lines for (a) the vertical reaction at $B$, (b) the shear just to the right of the rocker at $A$, and (c) the moment at $C$. Solve this problem using the basic method of Sec. 6-1.

(a)

(b)

(c)


6-6. Solve Prob. 6-5 using Müller-Breslau's principle.

(a)

(b)

(c)


6-7. Draw the influence line for (a) the moment at $B$, (b) the shear at $C$, and (c) the vertical reaction at $B$. Solve this problem using the basic method of Sec. 6-1. Hint: The support at $A$ resists only a horizontal force and a bending moment.

(a)

(b)

(C)

*6-8. Solve Prob. 6-7 using the Müller-Breslau principle.

(a)

(b)

(c)


6-9. Draw the influence line for (a) the vertical reaction at $A$, (b) the shear at $B$, and (c) the moment at $B$. Assume $A$ is fixed. Solve this problem using the basic method of Sec. 6-1.

(a)

(b)

(c)


6-10. Solve Prob. 6-9 using the Müller-Breslau principle.

(a)

(b)

(c)


6-11. Draw the influence lines for (a) the vertical reaction at $A$, (b) the shear at $C$, and (c) the moment at $C$. Solve this problem using the basic method of Sec. 6-1.

(a)

(b)

(c)

*6-12. Solve Prob. 6-11 using Müller-Breslau's principle.


(b)

(c)


6-13. Draw the influence lines for (a) the vertical reaction at $A$, (b) the vertical reaction at $B$, (c) the shear just to the right of the support at $A$, and (d) the moment at $C$. Assume the support at $A$ is a pin and $B$ is a roller. Solve this problem using the basic method of Sec. 6-1.

(a)

(b)

(C)

(d) $M_{c}$


6-14. Solve Prob. 6-13 using the Müller-Breslau principle.


(b) $B y$
(C)

(d) $M_{c}$


6-15. The beam is subjected to a uniform dead load of $1.2 \mathrm{kN} / \mathrm{m}$ and a single live load of 40 kN . Determine (a) the maximum moment created by these loads at $C$, and (b) the maximum positive shear at $C$. Assume $A$ is a pin. and $B$ is a roller.
a.) $M_{c}$

$\left(M_{C}\right)_{\max }=40 \mathrm{kN}(3 \mathrm{~m})+1.2 \mathrm{kN} / \mathrm{m}\left(\frac{1}{2}\right)(12 \mathrm{~m})(3 \mathrm{~m})=141.6 \mathrm{kN} \cdot \mathrm{m}$
b.)

$\left(V_{C}\right)_{\max }=40\left(\frac{1}{2}\right)+1.2 \mathrm{kN} / \mathrm{m}\left[\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(6)+\frac{1}{2}\left(\frac{1}{2}\right)(6)\right]=20 \mathrm{kN}$


Ans.

Ans.
*6-16. The beam supports a uniform dead load of $500 \mathrm{~N} / \mathrm{m}$ and a single live concentrated force of 3000 N . Determine (a) the maximum positive moment at $C$, and (b) the maximum positive shear at $C$. Assume the support at $A$ is a roller and $B$ is a pin.


Referring to the influence line for the moment at $C$ shown in Fig. $a$, the maximum positive moment at $C$ is

$$
\begin{aligned}
\left(M_{c}\right)_{\max (+)} & =0.75(3000)+\left[\frac{1}{2}(4-0)(0.75)\right](500) \\
& =3000 \mathrm{~N} \cdot \mathrm{~m}=3 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$

Ans.

Referring to the influence line for the moment at $C$ in Fig. $b$, the maximum positive shear at $C$ is

$$
\begin{aligned}
\left(V_{c}\right)_{\max (+)} & =0.75(3000)+\left[\frac{1}{2}(1-0)(-0.25)\right](500)+\left[\frac{1}{2}(4-1)(0.75)\right](500) \\
& =2750 \mathrm{~N}=2.75 \mathrm{kN}
\end{aligned}
$$


(a)

(b)

6-17. A uniform live load of $300 \mathrm{lb} / \mathrm{ft}$ and a single live concentrated force of 1500 lb are to be placed on the beam. The beam has a weight of $150 \mathrm{lb} / \mathrm{ft}$. Determine (a) the maximum vertical reaction at support $B$, and (b) the maximum negative moment at point $B$. Assume the support at $A$ is a pin and $B$ is a roller.


Referring to the influence line for the vertical reaction at $B$ shown in Fig. $a$, the maximum reaction that is

$$
\begin{aligned}
\left(B_{y}\right)_{\max (+)} & =1.5(1500)+\left[\frac{1}{2}(30-0)(1.5)\right](300+150) \\
& =12375 \mathrm{lb}=12.4 \mathrm{k}
\end{aligned}
$$

Ans.

Referring to the influence line for the moment at $B$ shown in Fig. $b$, the maximum negative moment is
$\left(M_{B}\right)_{\max (-)}=-10(1500)+\left[\frac{1}{2}(30-20)(-10)\right](300+150)$

$$
=-37500 \mathrm{lb} \cdot \mathrm{ft}=-37.5 \mathrm{k} \cdot \mathrm{ft}
$$

Ans.

(a)

(b)

6-18. The beam supports a uniform dead load of $0.4 \mathrm{k} / \mathrm{ft}$, a live load of $1.5 \mathrm{k} / \mathrm{ft}$, and a single live concentrated force of 8 k . Determine (a) the maximum positive moment at $C$, and (b) the maximum positive vertical reaction at $B$. Assume $A$ is a roller and $B$ is a pin.


Referring to the influence line for the moment at $C$ shown in Fig. $a$, the maximum positive moment is

$$
\begin{aligned}
\left(M_{C}\right)_{\max (+)}= & 5(8)+\left[\frac{1}{2}(20-0)(5)\right](1.5)+\left[\frac{1}{2}(20-0)(5)\right](0.4) \\
& +\left[\frac{1}{2}(35-20)(-7.5)\right](0.4) \\
= & 112.5 \mathrm{k} \cdot \mathrm{ft}
\end{aligned}
$$

Ans.

Referring to the influence line for the vertical reaction at $B$ shown in Fig. $b$, the maximum positive reaction is

$$
\begin{aligned}
\left(B_{y}\right)_{\max (+)}= & 1(8)+\left[\frac{1}{2}(20-0)(1)\right](1.5)+\left[\frac{1}{2}(20-0)(1)\right](0.4) \\
& +\left[\frac{1}{2}(35-20)(-0.75)\right](0.4) \\
= & 24.75 \mathrm{k}
\end{aligned}
$$

Ans.

(a)

(b)

6-19. The beam is used to support a dead load of $0.6 \mathrm{k} / \mathrm{ft}$, a live load of $2 \mathrm{k} / \mathrm{ft}$ and a concentrated live load of 8 k . Determine (a) the maximum positive (upward) reaction at $A$, (b) the maximum positive moment at $C$, and (c) the maximum positive shear just to the right of the support at $A$. Assume the support at $A$ is a pin and $B$ is a roller.

Referring to the influence line for the vertical reaction at $A$ shown in Fig. $a$, the maximum positive vertical reaction is

$$
\begin{aligned}
\left(A_{y}\right)_{\max (+)}= & 1.5(8)+\left[\frac{1}{2}(30-0)(1.5)\right](2) \\
& +\left[\frac{1}{2}(30-0)(1.5)\right](0.6)+\left[\frac{1}{2}(35-30)(-0.25)\right](0.6) \\
= & 70.1 \mathrm{k}
\end{aligned}
$$



Ans.
Referring to the influence line for the moment at $C$ shown in Fig. $b$, the maximum positive moment is

$$
\begin{aligned}
\left(M_{c}\right)_{\max (+)}= & 5(8)+\left[\frac{1}{2}(30-10)(5)\right](2)+\left[\frac{1}{2}(10-0)(-5)\right](0.6) \\
& +\left[\frac{1}{2}(30-10)(5)\right](0.6)+\left[\frac{1}{2}(35-30)(-2.5)\right](0.6)
\end{aligned}
$$

$$
=151 \mathrm{k} \cdot \mathrm{ft}
$$

Ans.

Referring to the influence line for shear just to the right of $A$ shown in Fig. $c$, the maximum positive shear is

$$
\begin{aligned}
\left(V_{A^{+}}\right)_{\max (+)}= & 1(8)+\left[\frac{1}{2}(10-0)(0.5)\right](2) \\
& +\left[\frac{1}{2}(30-10)(1)\right](2) \\
& +\left[\frac{1}{2}(10-0)(0.5)\right](0.6)+\left[\frac{1}{2}(30-10)(1)\right](0.6) \\
& +\left[\frac{1}{2}(35-30)(-0.25)\right](0.6) \\
= & 40.1 \mathrm{k}
\end{aligned}
$$



Ans.

*6-20. The compound beam is subjected to a uniform dead load of $1.5 \mathrm{kN} / \mathrm{m}$ and a single live load of 10 kN . Determine (a) the maximum negative moment created by these loads at $A$, and (b) the maximum positive shear at $B$. Assume $A$ is a fixed support, $B$ is a pin, and $C$ is a roller.
$\left(M_{A}\right)_{\max }=1.5\left(\frac{1}{2}\right)(15)(-5)+10(-5)$

$$
=-106 \mathrm{kN} \cdot \mathrm{~m}
$$

$\left(V_{B}\right)_{\text {max }}=1.5\left(\frac{1}{2}\right)(10)(1)+10(1)$
$=17.5 \mathrm{kN}$


Ans.


Ans.


6-21. Where should a single $500-\mathrm{lb}$ live load be placed on the beam so it causes the largest moment at $D$ ? What is this moment? Assume the support at $A$ is fixed, $B$ is pinned, and $C$ is a roller.


At point $B:\left(M_{D}\right)_{\text {max }}=500(-8)=-4000 \mathrm{lb} \cdot \mathrm{ft}=-4 \mathrm{k} \cdot \mathrm{ft}$


Ans.

6-22. Where should the beam $A B C$ be loaded with a $300 \mathrm{lb} / \mathrm{ft}$ uniform distributed live load so it causes (a) the largest moment at point $A$ and (b) the largest shear at $D$ ? Calculate the values of the moment and shear. Assume the support at $A$ is fixed, $B$ is pinned and $C$ is a roller.

(a) $\left(M_{A}\right)_{\max }=\frac{1}{2}(36)(-16)(0.3)=-86.4 \mathrm{k} \cdot \mathrm{ft}$
(b) $\left(V_{D}\right)_{\text {max }}=\left[(1)(8)+\frac{1}{2}(1)(20)\right](0.3)=5.40 \mathrm{k}$

Ans.

Ans.


6-23. The beam is used to support a dead load of $800 \mathrm{~N} / \mathrm{m}$, a live load of $4 \mathrm{kN} / \mathrm{m}$, and a concentrated live load of 20 kN . Determine (a) the maximum positive (upward) reaction at $B$,(b) the maximum positive moment at $C$, and (c) the maximum negative shear at $C$. Assume $B$ and $D$ are pins.


Referring to the influence line for the vertical reaction at $B$, the maximum positive reaction is

$$
\begin{aligned}
\left(B_{y}\right)_{\max (+)} & =1.5(20)+\left[\frac{1}{2}(16-0)(1.5)\right](4)+\left[\frac{1}{2}(16-0)(1.5)\right](0.8) \\
& =87.6 \mathrm{kN}
\end{aligned}
$$

Ans.

(a) positive moment is

$$
\begin{aligned}
\left(M_{c}\right)_{\max (+)}= & 2(20)+\left[\frac{1}{2}(8-0)(2)\right](4)+\left[\frac{1}{2}(8-0)(2)\right](0.8) \\
& +\left[\frac{1}{2}(16-8)(-2)\right](0.8) \\
= & 72.0 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$

Referring to the influence line for the shear at $C$ shown in, the maximum negative shear is

$$
\begin{aligned}
\left(V_{C}\right)_{\max (-)}= & -0.5(20)+\left[\frac{1}{2}(4-0)(-0.5)\right](4) \\
& +\left[\frac{1}{2}(16-8)(-0.5)\right](4)+\left[\frac{1}{2}(4-0)(-0.5)\right](0.8) \\
& +\left[\frac{1}{2}(8-4)(0.5)\right](0.8)+\left[\frac{1}{2}(16-8)(-0.5)\right](0.8) \\
= & -23.6 \mathrm{kN}
\end{aligned}
$$


(b)

Ans.

Ans.

(c)
*6-24. The beam is used to support a dead load of $400 \mathrm{lb} / \mathrm{ft}$, a live load of $2 \mathrm{k} / \mathrm{ft}$, and a concentrated live load of 8 k . Determine (a) the maximum positive vertical reaction at $A$, (b) the maximum positive shear just to the right of the support at $A$, and (c) the maximum negative moment at $C$. Assume $A$ is a roller, $C$ is fixed, and $B$ is pinned.


Referring to the influence line for the vertical reaction at $A$ shown in Fig. $a$, the maximum positive reaction is
$\left(A_{y}\right)_{\max (+)}=2(8)+\left[\frac{1}{2}(20-0)^{7}(2)\right](2+0.4)=64.0 \mathrm{k}$
Ans.

(a)

Referring to the influence line for the shear just to the right to the support at $A$ shown in Fig. $b$, the maximum positive shear is

$$
\begin{aligned}
\left(V_{A^{+}}\right)_{\max (+)}= & 1(8)+\left[\frac{1}{2}(10-0)(1)\right](2+0.4) \\
& +\left[\frac{1}{2}(20-10)(1)\right](2+0.4) \\
= & 32.0 \mathrm{k}
\end{aligned}
$$

Ans.

(b)

Referring to the influence line for the moment at $C$ shown in Fig. $c$, the maximum negative moment is

$$
\begin{aligned}
\left(M_{C}\right)_{\max (-)}= & -15(8)+\left[\frac{1}{2}(35-10)(-15)\right](2)+\left[\frac{1}{2}(10-0)(15)\right](0.4) \\
& +\left[\frac{1}{2}(35-10)(-15)\right](0.4) \\
= & -540 \mathrm{k} \cdot \mathrm{ft}
\end{aligned}
$$


(c)

6-25. The beam is used to support a dead load of $500 \mathrm{lb} / \mathrm{ft}$, a live load of $2 \mathrm{k} / \mathrm{ft}$, and a concentrated live load of 8 k . Determine (a) the maximum positive (upward) reaction at $A$, (b) the maximum positive moment at $E$, and (c) the maximum positive shear just to the right of the support at $C$. Assume $A$ and $C$ are rollers and $D$ is a pin.


Referring to the influence line for the vertical reaction at $A$ shown in Fig. $a$, the maximum positive vertical reaction is
$\left(A_{y}\right)_{\max (+)}=1(8)+\left[\frac{1}{2}(10-0)(1)\right](2+0.5)=20.5 \mathrm{k}$
Ans.
Referring to the influence line for the moment at $E$ shown in Fig. $b$, the maximum positive moment is

$$
\left(M_{E}\right)_{\max (+)}=2.5(8)+\left[\frac{1}{2}(10-0)(2.5)\right](2+0.5)
$$

$$
=51.25 \mathrm{k} \cdot \mathrm{ft}
$$

Ans.
Referring to the influence line for the shear just to the right of support $C$, shown in
Fig. $c$, the maximum positive shear is

$$
\begin{aligned}
\left(V_{C^{+}}\right)_{\max (+)}= & 1(8)+\left[\frac{1}{2}(15-0)(1)\right](2+0.5) \\
& +\left[\frac{1}{2}(20-15)(1)\right](2+0.5) \\
= & 33.0 \mathrm{k}
\end{aligned}
$$

Ans.

(a)

(b)


6-26. A uniform live load of $1.8 \mathrm{kN} / \mathrm{m}$ and a single concentrated live force of 4 kN are placed on the floor beams. Determine (a) the maximum positive shear in panel $B C$ of the girder and (b) the maximum moment in the girder at $G$.


Referring to the influence line for the shear in panel $B C$ shown in Fig. $a$, the maximum positive shear is
$\left(V_{B C}\right)_{\max (+)}=1(4)+\left[\frac{1}{2}(1-0.5)(1)\right](1.8)+[(2.5-1)(1)](1.8)=7.15 \mathrm{kN} \quad$ Ans.
Referring to the influence line for the moment at $G$ Fig. $b$, the maximum negative moment is

$$
\begin{aligned}
\left(M_{G}\right)_{\max (-)}= & -1.75(4)\left[\frac{1}{2}(1-0.5)(-0.25)\right](1.8) \\
& +\left\{\frac{1}{2}(2.5-1)[-0.25+(-1.75)]\right\}(1.8) \\
= & -9.81 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$

Ans.


(b)

6-27. A uniform live load of $2.8 \mathrm{kN} / \mathrm{m}$ and a single concentrated live force of 20 kN are placed on the floor beams. If the beams also support a uniform dead load of $700 \mathrm{~N} / \mathrm{m}$, determine (a) the maximum positive shear in panel $B C$ of the girder and (b) the maximum positive moment in the girder at $G$.


Referring to the influence line for the shear in panel $B C$ as shown in Fig. $a$, the maximum position shear is

$$
\begin{aligned}
\left(V_{B C}\right)_{\max (+)}= & 0.6(20)+\left[\frac{1}{2}(7.5-1.875)(0.6)\right](2.8) \\
& +\left[\frac{1}{2}(1.875-0)(-0.2)\right](0.7)+\left[\frac{1}{2}(7.5-1.875)(0.6)\right](0.7)
\end{aligned}
$$

$$
=17.8 \mathrm{kN}
$$

Ans.
Referring to the influence line for the moment at $G$ shown in Fig. $b$, the maximum positive moment is

$$
\begin{aligned}
\left(M_{G}\right)_{\max (+)}= & 1.35(20)\left[\frac{1}{2}(1.5-0)(1.05)\right](2.8+0.7) \\
& +\left[\frac{1}{2}(3-1.5)(1.05+1.35)\right](2.8+0.7) \\
& +\left[\frac{1}{2}(7.5-3)(1.35)\right](2.8+0.7) \\
= & 46.7 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$



*6-28. A uniform live load of $2 \mathrm{k} / \mathrm{ft}$ and a single concentrated live force of 6 k are placed on the floor beams. If the beams also support a uniform dead load of $350 \mathrm{lb} / \mathrm{ft}$, determine (a) the maximum positive shear in panel $C D$ of the girder and (b) the maximum negative moment in the girder at $D$. Assume the support at $C$ is a roller and $E$ is a pin.


Referring to the influence line for the shear in panel $C D$ shown in Fig. $a$, the maximum positive shear is

$$
\begin{aligned}
\left(V_{C D}\right)_{\max (+)} & =1(6)+\left[\frac{1}{2}(6-0)(1)\right](2+0.35)+\left[\frac{1}{2}(12-6)(0.5)\right](2+0.35) \\
& =16.575 \mathrm{k}=16.6 \mathrm{k}
\end{aligned}
$$

Referring to the influence line for the moment at $D$ shown in Fig. $b$, the maximum negative moment is

$$
\begin{aligned}
M_{D(\max )}= & -3(6)+\left[\frac{1}{2}(6-0)(-3)\right](2)+\left[\frac{1}{2}(6-0)(-3)\right](0.35) \\
& +\left[\frac{1}{2}(12-6)(1.5)\right](0.35) \\
= & -37.575 \mathrm{k} \cdot \mathrm{ft}=-37.6 \mathrm{k} \cdot \mathrm{ft}
\end{aligned}
$$

Ans.

(a)


6-29. Draw the influence line for (a) the shear in panel $B C$ of the girder, and (b) the moment at $D$.


6-30. A uniform live load of $250 \mathrm{lb} / \mathrm{ft}$ and a single concentrated live force of 1.5 k are to be placed on the floor beams. Determine (a) the maximum positive shear in panel $A B$, and (b) the maximum moment at $D$. Assume only vertical reaction occur at the supports.


$$
\begin{aligned}
\left(V_{A B}\right)_{\max }= & \frac{1}{2}(50-18.33)(0.5)(0.250)+0.5(1.5)=2.73 \mathrm{k} \\
\left(M_{D}\right)_{\max }= & {\left[\frac{1}{2}(3.75)(15)+\frac{1}{2}(3.75+7.5)(10)+\frac{1}{2}(7.5+6.25)(10)\right.} \\
& \left.+\frac{1}{2}(6.25)(15)\right](0.250)+7.5(1.5) \\
= & 61.25 \mathrm{k} \cdot \mathrm{ft}
\end{aligned}
$$

Ans.


Ans.


6-31. A uniform live load of $0.6 \mathrm{k} / \mathrm{ft}$ and a single concentrated live force of 5 k are to be placed on the top beams. Determine (a) the maximum positive shear in panel $B C$ of the girder, and (b) the maximum positive moment at $C$. Assume the support at $B$ is a roller and at $D$ a pin.

a) $\quad\left(V_{B C}\right)_{\text {max }}=5(0.5)+\frac{1}{2}(0.5)(30)(0.6)=7 \mathrm{k}$

Ans.
b) $\quad\left(M_{C}\right)_{\max }=7.5(5)+0.6\left[\left(\frac{1}{2}\right)(30)(7.5)\right]=105 \mathrm{k} \cdot \mathrm{ft}$

## Ans.


*6-32. Draw the influence line for the moment at $F$ in the girder. Determine the maximum positive live moment in the girder at $F$ if a single concentrated live force of 8 kN moves across the top floor beams. Assume the supports for all members can only exert either upward or downward forces on the members.


Ans.


6-33. A uniform live load of $4 \mathrm{k} / \mathrm{ft}$ and a single concentrated live force of 20 k are placed on the floor beams. If the beams also support a uniform dead load of $700 \mathrm{lb} / \mathrm{ft}$, determine (a) the maximum negative shear in panel $D E$ of the girder and (b) the maximum negative moment in the girder at $C$.


By referring to the influence line for the shear in panel $D E$ shown in Fig. $a$, the maximum negative shear is

$$
\begin{aligned}
\left(V_{D E}\right)_{\max (-)}= & (-1)(20)+[(6-0)(-1)](4+0.7) \\
& +\left[\frac{1}{2}(8-6)(-1)\right](4+0.7) \\
= & -52.9 \mathrm{k}
\end{aligned}
$$

Ans.
By referring to the influence line for the moment at $C$ shown in Fig. $b$, the maximum negative moment is

$$
\begin{aligned}
\left(M_{C}\right)_{\max (-)} & =-4(20)+\left[\frac{1}{2}(4-0)(-4)\right](4+0.7) \\
& =-118 \mathrm{k} \cdot \mathrm{ft}
\end{aligned}
$$

Ans.

(a)


6-34. A uniform live load of $0.2 \mathrm{k} / \mathrm{ft}$ and a single concentrated live force of 4 k are placed on the floor beams. Determine (a) the maximum positive shear in panel $D E$ of the girder, and (b) the maximum positive moment at $H$.


Referring to the influence line for the shear in panel $D E$ shown in Fig. $a$, the maximum positive shear is

$$
\begin{aligned}
\left(V_{D E}\right)_{\max (+)}= & 0.6667(4)+\left[\frac{1}{2}(18-0)(0.6667)\right](0.2) \\
& +\left[\frac{1}{2}(36-18)(0.6667)\right](0.2) \\
= & 5.07 \mathrm{k}
\end{aligned}
$$

Ans.

Referring to the influence line for the moment at $H$ shown in Fig. $b$, the maximum positive moment is

$$
\begin{aligned}
\left(M_{H}\right)_{\max (+)}= & 3(4)+\left[\frac{1}{2}(24-18)(3)\right](0.2) \\
& +[(30-24)(3)](0.2)+\left[\frac{1}{2}(36-30)(3)\right](0.2) \\
= & 19.2 \mathrm{k} \cdot \mathrm{ft}
\end{aligned}
$$

Ans.

(a)

(b)

6-35. Draw the influence line for the shear in panel $C D$ of the girder. Determine the maximum negative live shear in panel $C D$ due to a uniform live load of $500 \mathrm{lb} / \mathrm{ft}$ acting on the top beams.

$\left(V_{C D}\right)_{\max (-)}=500\left(\frac{1}{2}\right)(32)(-0.75)=-6 \mathrm{k}$

*6-36. A uniform live load of $6.5 \mathrm{kN} / \mathrm{m}$ and a single concentrated live force of 15 kN are placed on the floor beams. If the beams also support a uniform dead load of $600 \mathrm{~N} / \mathrm{m}$, determine (a) the maximum positive shear in panel $C D$ of the girder and (b) the maximum positive


Referring to the influence line for the shear in panel $C D$ shown in Fig. $a$, the maximum positive shear is

$$
\begin{aligned}
\left(V_{C D}\right)_{\max (+)}= & (0.3333)(15)+\left[\frac{1}{2}(4-0)(0.3333)\right](6.5+0.6) \\
& +\left[\frac{1}{2}(16-10)(0.3333)\right](6.5+0.6) \\
& +\left[\frac{1}{2}(10-4)(-0.3333)\right](0.6) \\
= & 16.2 \mathrm{kN}
\end{aligned}
$$

## Ans.

Referring to the influence line for the moment at $D$ shown in Fig. $b$, the maximum positive moment is

$$
\begin{aligned}
\left(M_{D}\right)_{\max (+)}= & 2.6667(15)+\left[\frac{1}{2}(16-4)(2.6667)\right](6.5+0.6) \\
& +\left[\frac{1}{2}(4-0)(-1.3333)\right](0.6) \\
= & 152 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$


(a)

Ans.

(b)

6-37. A uniform live load of $1.75 \mathrm{kN} / \mathrm{m}$ and a single concentrated live force of 8 kN are placed on the floor beams. If the beams also support a uniform dead load of $250 \mathrm{~N} / \mathrm{m}$, determine (a) the maximum negative shear in panel $B C$ of the girder and (b) the maximum positive moment at $B$.


By referring to the influence line for the shear in panel $B C$ shown in Fig. $a$, the maximum negative shear is
$\left(V_{B C}\right)_{\max (-)}=-0.6667(8)$

$$
\begin{aligned}
& +\left[\frac{1}{2}(4.5-0)(-0.6667)\right](1.75+0.25) \\
& +\left[\frac{1}{2}(6-4.5)(0.6667)\right](0.25) \\
= & -8.21 \mathrm{kN}
\end{aligned}
$$

Ans.
By referring to the influence line for the moment at $B$ shown in Fig. $b$, the maximum positive moment is

$$
\begin{aligned}
\left(M_{B}\right)_{\max (+)}= & 1(8)+\left[\frac{1}{2}(4.5-0)(1)\right](1.75+0.25) \\
& +\left[\frac{1}{2}(6-4.5)(-1)\right](0.25) \\
= & 12.3 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$


(a)

(b)

6-38. Draw the influence line for the force in (a) member $K J$ and (b) member $C J$.





6-39. Draw the influence line for the force in (a) member $J I$, (b) member $I E$, and (c) member $E F$.

$\vdash 6 \mathrm{ft} \rightarrow 6 \mathrm{ft} \rightarrow 6 \mathrm{ft} \rightarrow 6 \mathrm{ft} \rightarrow 6 \mathrm{ft} \rightarrow 6 \mathrm{ft} \rightarrow$

$F_{\text {IE }}$

*6-40. Draw the influence line for the force in member $K J$.


6-41. Draw the influence line for the force in member $J E$.


6-42. Draw the influence line for the force in member $C D$.



6-43. Draw the influence line for the force in member $J K$.

*6-44. Draw the influence line for the force in member $D K$.


6-45. Draw the influence line for the force in (a) member $E H$ and (b) member $J E$.



6-46. Draw the influence line for the force in member $J I$.


6-47. Draw the influence line for the force in member $A L$.

*6-48. Draw the influence line for the force in member $B C$ of the Warren truss. Indicate numerical values for the peaks. All members have the same length.


6-49. Draw the influence line for the force in member $B F$ of the Warren truss. Indicate numerical values for the peaks. All members have the same length.


6-50. Draw the influence line for the force in member $F E$ of the Warren truss. Indicate numerical values for the peaks. All members have the same length.

*6-52. Draw the influence line for the force in member $D L$.


6-53. Draw the influence line for the force in member $C D$.



6-54. Draw the influence line for the force in member $C D$.


6-55. Draw the influence line for the force in member $K J$.

*6-56. Draw the influence line for the force in member $G D$, then determine the maximum force (tension or compression) that can be developed in this member due to a uniform live load of $3 \mathrm{kN} / \mathrm{m}$ that acts on the bridge deck along the bottom cord of the truss.


Referring to the influence line for the member force of number $G D$, the maximum tensile and compressive force is
$\left(F_{G D}\right)_{\max (+)}=\left[\frac{1}{2}(12-6.857)(0.751)\right](3)=5.79 \mathrm{kN}(\mathrm{T})($ Max. $)$
$\left(F_{G D}\right)_{\min (-)}=\left[\frac{1}{2}(6.857-0)(-0.300)\right](3)=-3.09 \mathrm{kN}(\mathrm{C})$


6-57. Draw the influence line for the force in member $C D$, and then determine the maximum force (tension or compression) that can be developed in this member due to a uniform live load of $800 \mathrm{lb} / \mathrm{ft}$ which acts along the bottom cord of the truss.


Referring to the influence line for the force of member $C D$, the maximum tensile force is
$\left(F_{C D}\right)_{\max (+)}=\left[\frac{1}{2}(40-0)(0.75)\right](0.8)=12.0 \mathrm{k}(\mathrm{T})$
Ans.


6-58. Draw the influence line for the force in member $C F$, and then determine the maximum force (tension or compression) that can be developed in this member due to a uniform live load of $800 \mathrm{lb} / \mathrm{ft}$ which is transmitted to the truss along the bottom cord.


Referring to the influence line for the force in member $C F$, the maximum tensile and compressive force are

$$
\begin{aligned}
\left(F_{C F}\right)_{\max (+)} & =\left[\frac{1}{2}(26.67-0)(0.7071)\right](0.8)=7.54 \mathrm{k}(\mathrm{~T}) \\
\left(F_{C F}\right)_{\max (-)} & =\left[\frac{1}{2}(40-26.67)(-0.3536)\right](0.8) \\
& =-1.89 \mathrm{k}=1.89 \mathrm{k}(\mathrm{C})
\end{aligned}
$$



6-59. Determine the maximum live moment at point $C$ on the single girder caused by the moving dolly that has a mass of 2 Mg and a mass center at $G$. Assume $A$ is a roller.


$$
\left(M_{C}\right)_{\max }=14.715(2.5)+4.905(1.5)=44.1 \mathrm{kN} \cdot \mathrm{~m}
$$

Ans.

*6-60. Determine the maximum live moment in the suspended rail at point $B$ if the rail supports the load of 2.5 k on the trolley.

Check maximum positive moment:
$\left(M_{B}\right)_{\text {max }}=1.667(3)+(0.833)(1.5)=6.25 \mathrm{k} \cdot \mathrm{ft}$


Check maximum negative moment:

$$
\frac{h}{5}=\frac{4}{8} ; \quad h=2.5 \mathrm{ft}
$$

$$
\left(M_{B}\right)_{\max }=1.667(-4)+(0.833)(-2.5)=-8.75 \mathrm{k} \cdot \mathrm{ft}
$$




6-61. Determine the maximum positive shear at point $B$ if the rail supports the load of 2.5 k on the trolley.

The position for maximum positive shear is shown.


$$
\frac{h}{5}=\frac{0.667}{8} ; \quad h=0.41667
$$

$$
\left(V_{B}\right)_{\max }=1.667(0.667)+0.833(0.41667)=1.46 \mathrm{k}
$$

## Ans.



6-62. Determine the maximum positive moment at the splice $C$ on the side girder caused by the moving load which travels along the center of the bridge.


The maximum positive moment at point $C$ occurs when the moving loads are at the position shown in Fig. $a$.
$\left(M_{C}\right)_{\max (+)}=4(4)+2(2)=20.0 \mathrm{kN} \cdot \mathrm{m}$
Ans.


6-63. Determine the maximum moment at $C$ caused by the moving load.


The vertical reactions of the wheels on the girder are as shown in Fig. a. The maximum positive moment at point $C$ occurs when the moving loads are at the position shown in Fig. $b$.
$\left(M_{C}\right)_{\max (+)}=7.5(1600)+6(800)=16800 \mathrm{lb} \cdot \mathrm{ft}$

$$
=16.8 \mathrm{k} \cdot \mathrm{ft}
$$

Ans.


(b)
*6-64. Draw the influence line for the force in member $I H$ of the bridge truss. Determine the maximum force (tension or compression) that can be developed in this member due to a $72-\mathrm{k}$ truck having the wheel loads shown. Assume the truck can travel in either direction along the center of the deck, so that half its load is transferred to each of the two side trusses. Also assume the members are pin-connected at the gusset plates.
$\left(F_{I H}\right)_{\max }=0.75(4)+16(0.7)+16(1.2)=33.4 \mathrm{k}(\mathrm{C})$


Ans.


6-65. Determine the maximum positive moment at point $C$ on the single girder caused by the moving load.


Move the $8-\mathrm{kN}$ force 2 m to the right of $C$. The change in moment is
$\Delta M=8\left(-\frac{2.5}{5}\right)(2 \mathrm{~m})+6\left(\frac{2.5}{5}\right)(2)+4\left(\frac{2.5}{5}\right)(2)=2 \mathrm{kN} \cdot \mathrm{m}$
Since $\Delta M$ is positive, we must investigate further. Next move the 6 kN force 1.5 m to the right of $C$, the change in moment is
$\Delta M=8\left(-\frac{2.5}{5}\right)(1.5)+6\left(-\frac{2.5}{5}\right)(1.5)+4\left(\frac{2.5}{5}\right)(1.5)=-7.5 \mathrm{kN} \cdot \mathrm{m}$
Since $\Delta M$ is negative, the case where the 6 kN force is at $C$ will generate the maximum positive moment, Fig. a.
$\left(M_{C}\right)_{\max (+)}=1.75(4)+6(2.5)+8(1.5)=34.0 \mathrm{kN} \cdot \mathrm{m}$
Ans.

(a)

6-66. The cart has a weight of 2500 lb and a center of gravity at $G$. Determine the maximum positive moment created in the side girder at $C$ as it crosses the bridge. Assume the car can travel in either direction along the center of the deck, so that half its load is transferred to each of the two side girders.


The vertical reaction of wheels on the girder are indicated in Fig. $a$. The maximum positive moment at point $C$ occurs when the moving loads are in the positions shown in Fig. $b$. Due to the symmetry of the influence line about $C$, the maximum positive moment for both directions are the same.
$\left(M_{C}\right)_{\max (+)}=4(750)+2.75(500)=4375 \mathrm{lb} \cdot \mathrm{ft}=4.375 \mathrm{k} \cdot \mathrm{ft}$
Ans.

(a)

(b)

6-67. Draw the influence line for the force in member $B C$ of the bridge truss. Determine the maximum force (tension or compression) that can be developed in the member due to a 5 -k truck having the wheel loads shown. Assume the truck can travel in either direction along the center of the deck, so that half the load shown is transferred to each of the two side trusses. Also assume the members are pin connected at the gusset plates.

$\left(F_{B C}\right)_{\max }=\frac{3(1)+2(0.867)}{2}=2.37 \mathrm{k}(\mathrm{T})$
Ans.

*6-68. Draw the influence line for the force in member IC of the bridge truss. Determine the maximum force (tension or compression) that can be developed in the member due to a 5 -k truck having the wheel loads shown. Assume the truck can travel in either direction along the center of the deck, so that half the load shown is transferred to each of the two side trusses. Also assume the members are pin connected at the gusset plates.

$\left(F_{I C}\right)_{\max }=\frac{3(0.833)+2(0.667)}{2}=1.92 \mathrm{k}(\mathrm{T})$
Ans.


6-69. The truck has a mass of 4 Mg and mass center at $G_{1}$, and the trailer has a mass of 1 Mg and mass center at $G_{2}$. Determine the absolute maximum live moment developed in the bridge.

Loading Resultant Location
$\bar{x}=\frac{9810(0)+39240(3)}{49050}=2.4 \mathrm{~m}$
One possible placement on bridge is shown in FBD (1),
From the segment (2):
$M_{\max }=27284(4.45)-26160(2.25)=62.6 \mathrm{kN} \cdot \mathrm{m}$
Another possible placement on bridge is shown in Fig. (3),
From the segment (4):
$M_{\max }=20386.41(3.325)=67.8 \mathrm{kN} \cdot \mathrm{m}$

(2)


9810 N I3040N

Ans.

(4)

6-70. Determine the absolute maximum live moment in the bridge in Problem 6-69 if the trailer is removed.


Placement is shown in FBD (1). Using segment (2):
$M_{\max }=17780.625(3.625)=64.5 \mathrm{kN} \cdot \mathrm{m}$


6-71. Determine the absolute maximum live shear and absolute maximum moment in the jib beam $A B$ due to the $10-\mathrm{kN}$ loading. The end constraints require $0.1 \mathrm{~m} \leq x \leq 3.9 \mathrm{~m}$.

Abs. max. shear occurs when $0.1 \leq x \leq 3.9 \mathrm{~m}$

$$
V_{\max }=10 \mathrm{kN}
$$

Abs. max. moment occurs when $x=3.9 \mathrm{~m}$
$M_{\max }=-10(3.9)=-39 \mathrm{kN} \cdot \mathrm{m}$

Ans.


Ans.
*6-72. Determine the maximum live moment at $C$ caused by the moving loads.


The worst case is

$$
\left(M_{C}\right)_{\max }=2(10.2)+4(12.0)+6(10.4)+2(9.2)=149 \mathrm{k} \cdot \mathrm{ft}
$$

Ans.


6-73. Determine the absolute maximum moment in the girder bridge due to the truck loading shown. The load is applied directly to the girder.

$C+\sum M_{\max }=0 ; \quad M_{\max }+10(20)-18.175(41.543)=0$

$$
M_{\max }=555 \mathrm{k} \cdot \mathrm{ft}
$$

Ans.


6-74. Determine the absolute maximum shear in the beam due to the loading shown.


The maximum shear occurs when the moving loads are positioned either with the 40 kN force just to the right of the support at $A$, Fig. $a$, or with the 20 kN force just to of the support it $B$, Fig. $b$. Referring to Fig. $a$,

$$
\begin{array}{cl}
C+\sum M_{B}=0 ; & 40(12)+25(8)+20(6.5)-A_{y}(12)=0 \\
A_{y}=67.5 \mathrm{kN}
\end{array}
$$

Referring to Fig. $b$,

$$
\begin{gathered}
S+\sum M_{A}=0 ; B_{y}(12)-20(12)-25(10.5)-40(6.5)=0 \\
B_{y}=63.54 \mathrm{kN}
\end{gathered}
$$

Therefore, the absolute maximum shear occurs for the case in Fig. $a$,

$$
V_{\text {abs }}^{\substack{\max }}=A_{y}=67.5 \mathrm{kN}
$$

Ans.

(a)

(b)

6-75. Determine the absolute maximum moment in the beam due to the loading shown.

Referring to Fig. $a$, the location of $F_{R}$ for the moving load is

$+\downarrow F_{R}=\sum F_{y} ; \quad F_{R}=40+25+20=85 \mathrm{kN}$
$\zeta+F_{R} \bar{x}=\sum M_{C} ; \quad-85 \bar{x}=-25(4)-20(5.5)$
$\bar{x}=2.4706 \mathrm{~m}$
Assuming that the absolute maximum moment occurs under 40 kN force, Fig. $b$.
$\zeta+\sum M_{B}=0 ; \quad 20(1.7353)+25(3.2353)+40(7.2353)-A_{y}(12)=0$

$$
A_{y}=33.75 \mathrm{kN}
$$

Referring to Fig. $c$,

$$
\begin{aligned}
C+\sum M_{S}=0 ; & M_{S}-33.75(4.7647)=0 \\
& M_{S}=160.81 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$

Assuming that the absolute moment occurs under 25 kN force, Fig. $d$.

$$
\begin{gathered}
C+\sum M_{A}=0 ; \quad B_{y}(12)-40(2.7647)-25(6.7647)-20(8.2647)=0 \\
B_{y}=37.083 \mathrm{kN}
\end{gathered}
$$

Referring to Fig. $e$,
$\zeta+\sum M_{S}=0 ; \quad 37.083(5.2353)-20(1.5)-M_{S}=0$

$$
M_{S}=164.14 \mathrm{kN} \cdot \mathrm{~m}=164 \mathrm{kN} \cdot \mathrm{~m}(\text { Abs. Max. })
$$



*6-76. Determine the absolute maximum shear in the bridge girder due to the loading shown.


By inspection the maximum shear occurs when the moving loads are position with the 10 k force just to the left of the support at $B$, Fig. $b$.

$$
\begin{gathered}
C+\sum M_{A}=0 ; \quad B_{y}(30)-6(22)-10(30)=0 \\
B_{y}=14.4 \mathrm{k}
\end{gathered}
$$

Therefore, the absolute maximum shear is


6-77. Determine the absolute maximum moment in the bridge girder due to the loading shown.

Referring to Fig. $a$, the location of $F_{R}$ for the moving load is


$$
\begin{array}{lll}
+\downarrow F_{R}=\sum F_{y} ; & -F_{R}=-6-10 & F_{R}=16 \mathrm{k} \\
\zeta+F_{R} \bar{x}=\sum M_{C} ; & -16 \bar{x}=-10(8) \quad \bar{x}=5 \mathrm{ft} &
\end{array}
$$

By observation, the absolute maximum moment occurs under the $10-\mathrm{k}$ force, Fig. $b$,
$C+\sum M_{A}=0 ; \quad B_{y}(30)-6(8.5)-10(16.5)=0 \quad B_{y}=7.20 \mathrm{k}$
Referring to Fig. $c$,

$\varsigma+\sum M_{S}=0 ; 7.20(13.5)-M_{S}=0 \quad M_{S}=97.2 \mathrm{k} \cdot \mathrm{ft}($ Abs. Max. $)$


6-78. Determine the absolute maximum moment in the girder due to the loading shown.


Referring to Fig. $a$, the location of $F_{R}$ for the moving load is

$$
\begin{array}{cc}
+\downarrow F_{R}=\sum F_{y} ; & F_{R}=10+8+3+4=25 \mathrm{k} \\
C+F_{R} \bar{x}=\sum M_{C} ; & -25 \bar{x}=-8(3)-3(5)-4(7) \\
\bar{x}=2.68 \mathrm{ft} .
\end{array}
$$

Assuming that the absolute maximum moment occurs under 10 k load, Fig. $b$,

$$
\begin{gathered}
\varsigma+\sum M_{B}=0 ; \quad 4(6.84)+3(8.84)+8(10.84)+10(13.84)-A_{y}(25)=0 \\
A_{y}=11.16 \mathrm{k}
\end{gathered}
$$



Referring to Fig. $c$,

$$
\begin{aligned}
C+\sum M_{S}=0 ; & M_{S}-11.16(11.16)=0 \\
& M_{S}=124.55 \mathrm{k} \cdot \mathrm{ft}
\end{aligned}
$$

Assuming that the absolute maximum moment occurs under the 8-k force, Fig. $d$,

$$
\begin{gathered}
S+\sum M_{B}=0 ; \quad 4(8.34)+3(10.34)+8(12.34)+10(15.34)-A_{y}(25)=0 \\
A_{y}=12.66 \mathrm{k}
\end{gathered}
$$

Referring to Fig. $e$,

$$
\begin{aligned}
\mathrm{C}+\sum M_{S}=0 ; & M_{S}+10(3)-12.66(12.66)=0 \\
& M_{S}=130.28 \mathrm{k} \cdot \mathrm{ft}=130 \mathrm{k} \cdot \mathrm{ft}(\text { Abs. Max. })
\end{aligned}
$$



Ans.

(e)

$A_{y}=12.66 \mathrm{~K}$

6-79. Determine the absolute maximum shear in the beam due to the loading shown.


The maximum shear occurs when the moving loads are positioned either with the 3-k force just to the right of the support at $A$, Fig. $a$, or with the 4 k force just to the left of the support at $B$. Referring to Fig. $a$
$\zeta+\sum M_{B}=0 ; \quad 4(19)+2(22)+6(25)+3(30)-A_{y}(30)=0$

$$
A_{y}=12.0 \mathrm{k}
$$

Referring to Fig. $b$
$\zeta+\sum M_{A}=0 ; \quad B_{y}(30)-3(19)-6(24)-2(27)-4(30)=0$

$$
B_{y}=12.5 \mathrm{k}
$$

Therefore, the absolute maximum shear occurs for the case in Fig. $b$

$$
V_{\substack{\text { abs } \\ \max }}=B_{y}=12.5 \mathrm{k}
$$

Ans.

*6-80. Determine the absolute maximum moment in the bridge due to the loading shown.


Referring to Fig. $a$, the location of the $F_{R}$ for the moving loads is

$$
\begin{gathered}
+\downarrow F_{R}=\sum F_{y} ; \quad F_{R}=3+6+2+4+15 \mathrm{k} \\
C+F_{R} \bar{x}=\sum M_{C} ; \\
\bar{x}=6 \mathrm{ft}
\end{gathered}
$$

## *6-80. Continued

Assuming that the absolute maximum moment occurs under the 6 k force, Fig. $b$,

$$
\begin{gathered}
C+\sum M_{B}=0 ; \quad 4(9.5)+2(12.5)+6(15.5)+3(20.5)-A_{y}(30)=0 \\
A_{y}=7.25 \mathrm{k}
\end{gathered}
$$

Referring to Fig. $c$,

$$
\zeta+\sum M_{S}=0 ; \quad M_{S}+3(5)-7.25(14.5)=0
$$

$$
M_{S}=90.1 \mathrm{k} \cdot \mathrm{ft}
$$

Ans.
Assuming that the absolute maximum moment occurs under the 2 k force, Fig. $d$,

$$
\begin{gathered}
\varsigma+\sum M_{A}=0 ; \quad B_{y}(30)-3(8)-6(13)-2(16)-4(19)=0 \\
B_{y}=7.00 \mathrm{k}
\end{gathered}
$$

Referring to Fig.e,
$\zeta+\sum M_{S}=0 ; \quad 7.00(14)-4(3)-M_{S}=0$

$$
M_{S}=86.0 \mathrm{k} \cdot \mathrm{ft}(\mathrm{Abs} . \text { Max. })
$$


(b)


(a)


6-81. The trolley rolls at $C$ and $D$ along the bottom and top flange of beam $A B$. Determine the absolute maximum moment developed in the beam if the load supported by the trolley is 2 k . Assume the support at $A$ is a pin and at $B$ a roller.

Referring to the FBD of the trolley in Fig. $a$,

$\zeta+\sum M_{C}=0 ;$

$$
N_{D}(1)-2(1.5)=0
$$

$$
N_{D}=3.00 \mathrm{k}
$$

$$
\varsigma+\sum M_{D}=0 ; \quad N_{C}(1)-2(0.5)=0 \quad N_{C}=1.00 \mathrm{k}
$$

Referring to Fig. $b$, the location of $F_{R}$

$$
\begin{array}{ll}
+\downarrow F_{R}=\sum F_{Y} ; & F_{R}=3.00-1.00=2.00 \mathrm{k} \downarrow \\
\varsigma+F_{R} \bar{x}=\sum M_{C} ; & -2.00(\bar{x})=-3.00(1) \\
& \bar{x}=1.5 \mathrm{ft}
\end{array}
$$

The absolute maximium moment occurs under the 3.00 k force, Fig. $c$.

$$
\begin{gathered}
\varsigma+\sum M_{A}=0 ; \quad B_{y}(20)+1.00(8.75)-3.00(9.75)=0 \\
B_{y}=1.025 \mathrm{k}
\end{gathered}
$$

Referring to Fig. $d$,

$$
\begin{aligned}
\zeta+\sum M_{S}=0 ; & 1.025(10.25)-M_{S}=0 \\
& M_{S}=10.5 \mathrm{k} \cdot \mathrm{ft}(\text { Abs. Max. })
\end{aligned}
$$



Ans.


